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Modelling and identification of switched reluctance machine inductance

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**ARTICLE**

**ABSTRACT**

Identification of phase self-inductance is a central step in control and development of Switched Reluctance Machine (SRM) model because the voltage and torque equation are functions of the phase self-inductance. The SRM operates primarily in the saturation region because the power factor is lower in the linear region, which makes the SRM modelling difficult to achieve. The strong nonlinearity of this machine must be taken into consideration by appropriate identification of the inductance parameters. This paper presents a new method for analytical modelling and identification of nonlinear inductance at standstill tests. We have shown that our electrical model of SRM can be described by Hammerstein model as a typical nonlinear model with a specific structure. The proposed method differs substantially from the existing used to identify the inductance of SRM. The results of this analytical approach are compared with those obtained using magnetostatic finite element method and experimental measurement. The comparison results show good accuracy of the identified model.

**1. Introduction**

The switched reluctance motor was one of the first invented electrical devices (Miller 1993). Despite that, the true potential of the machine was inappropriate to be used with the mechanical switches that were available at that time (Miller 1993). The SRM has re-emerged and came into practical use in the 1970s due to the fast development and availability of power electronic devices (Miller 1993). Nowadays, the SRM has attracted significant attention, because it has several advantages compared to the conventional engines (Terzic et al. 2018). Firstly, there is no permanent magnets (free of rare-earth) unlike permanent magnet devices and rotor winding in contrast to induction machines, which reduces the electric motor production cost and facilitates recycling (Chiba et al. 2011; Bilgin, Jiang, and Emadi 2019). Furthermore, the cooling is simpler (Viajante et al. 2013) because losses show up mainly in the stator. Secondly, each phase is independent electrically and magnetically from the other phases, which increases its reliability in fault situations and makes its maintenance much easier (Soares and Costa Branco 2001). Thirdly, the SRM is characterised by high speeds operation (Soares and Costa Branco 2001). However, SRM suffers from high torque ripples, vibration and noise (Chiba et al. 2011; Soares and Costa Branco 2001), several studies are underway to reduce these disadvantages (Inanc and Ozbulur 2003). In order to formulate speed, torque and DC-link voltage controller, it is necessary to have accurate electromagnetic characteristics (flux linkage, self-inductance) of the machine. Therefore, these characteristics are highly saturated which make the linear model unsuitable and the development of controls among most challenging (Barros et al. 2018; Kadi and Brouri 2017). Consequently, the nonlinear electromagnetic characteristics of SRM must be identified as a function of the phase current and rotor position (Barros et al. 2018).

Even though the construction of SRM is simple, the strong nonlinearity makes the modelling of SRM not an easy task. Previous research has shown that two characteristics have been used in SRM modelling (Emadi 2017) as shown in Figure 1. The first method is established using an inductance-based model (Wenzhe, Keyhani, and Fardoun 2003). The second solution is based on the flux linkage characteristics (Wenzhe, Keyhani, and Fardoun 2003). Several researchers have addressed the determination problem of either of the two electromagnetic characteristics (Terzic et al. 2018; Fahimi et al. 1998, May; Stiebler and Liu 1999; Radun 2000; Ustun 2009; Wang 2012; Aguado-Rojas, Maya-Ortiz, and Espinosa-Pérez 2018). Different methods exist in the literature regarding the modelling and the identification of SRM flux linkage and self-inductance characteristics. The commonly used are experimental measurements using Lookup Table (Terzic et al. 2018;
Barros et al. (2018), Numerical method based on magnetostatic and transient finite element analysis (FEA) (Kadi and Brouri 2017; Parreira et al. 2005; Ganji, Heidarian, and Faiz 2015), intelligent methods such as artificial neural networks and fuzzy logic system (Ustun 2009; Yao and Yang 2017) and analytical methods (Khalil and Husain 2005; Li et al. 2016). Look-up tables use generally mathematical interpolation techniques. In (Barros et al. 2018), a smoothing spline interpolation is suggested to express the nonlinear electromagnetic characteristics of SRM, this method allows to obtain an accurate model of electromagnetic characteristic. However, a high number of data are required to perform these characteristics. In (Ustun 2009; Yao and Yang 2017), the authors proposed neural network and fuzzy-based models. These methods are appropriate for dealing with nonlinearities but are not practical to design the controller due to its complexity. By neglecting the saturation effect the authors in (Aguado-Rojas, Maya-Ortiz, and Espinosa-Pérez 2018) suggest on-line estimation of SRM parameters. Notwithstanding in this work, the identification method deals on-line, but it processes by neglecting the saturation effect. This assumption is unsuitable for this kind of electrical machine because the co-energy rises with the level of saturation as a result it operates mainly in the magnetically saturated region (Bilgin, Jiang, and Emadi 2019; Khalil 2005), so the determined model could be useful only in the low current region.

To develop an accurate nonlinear analytical model for SRM is one of the main motivation of our work. Furthermore, the proposed model must be simple as possible and characterised by a reduced computational time. Many applications of block-oriented nonlinear models which consist of linear dynamic subsystem and memory-less nonlinear elements such as Hammerstein, Wiener and Hammerstein-Wiener models have been presented in the literature for a variety of nonlinear systems (Balestrino et al. 2001; Brouri et al. 2014b, 2014a; Giri et al. 2014) To the author’s knowledge, for the first time it has been shown that the SRM can be described by a Hammerstein model. In the present paper, using the inductance model, we propose a mathematically practical technique with no computation complexity and good accuracy to identify the inductance parameters of SRM. Indeed, we show that SRM at standstill can be described by the cascade connection of a static nonlinearity block followed by a linear dynamic system (Figure 2). This nonlinear system is called Hammerstein model (Brouri et al. 2011a). As soon as a model is selected, the parameters identification becomes an important problem. Presently, we propose an identification algorithm based on Recursive Least Squares (RLS) method to determine the inductance parameters of SRM. However, the SRM can be modelled analytically using data acquisition obtained by experiments or by using the FEM (Finite Element Method) technique (Bilgin, Jiang, and Emadi 2019; Barros et al. 2018). Accordingly, in the proposed standstill experience, the SRM is excited by a signal (current source) having rich frequency spectrum (e.g. multisine signal). Then, knowing the nonlinear model describing the SRM (Hammerstein structure), its parameters can be determined using data acquisition (i.e. current and voltage winding values) and recursive least square (RLS) algorithm. Recall that, the curve \( L(i) \) for any given rotor position \( \theta \) is continuous. According to the Weierstrass approximation theorem, \( L(i) \) can be described involving basic functions decomposition or any complete orthogonal functions (e.g. polynomial basis) (Hur, Kim, and Hyun 2003; Suresh et al. 1999). In this study, we propose a method using polynomial function decomposition of \( L(i) \) (i.e. the variation of phase inductance at different position \( \theta \) according to phase current \( i \)).

Unlike many of previous works (Aguado-Rojas, Maya-Ortiz, and Espinosa-Pérez 2018; Hur, Kim, and Hyun 2003; Suresh et al. 1999; Peng, Cassani, and Williamson 2010), a novel method of analytical identification of SRM is given in this paper based on block-oriented nonlinear models. Finally, note that this approach also enjoys the simplicity of implementation, good convergence properties, and very fast computing time.

The paper is organised as follows: The operation principle of switched reluctance machine studied is introduced in Section 2. The electrical equations describing operating of the machines are dealt with in Section 3. The steps of inductance parameters determination algorithm of switched reluctance machine are discussed in section 4. Finally, simulation examples are carried out to verify the effectiveness of the proposed method are coped within Section 5.

![Figure 1](image1.png)

Figure 1. Classification of SRM modelling.

![Figure 2](image2.png)

Figure 2. Hammerstein nonlinear model.
2. Operation principle of SRM

Switched reluctance machine is characterized by a passive rotor (a salient ferromagnetic circuit with no conductor nor magnet) unlike induction and permanent magnet devices. The stator also comprises a salient ferromagnetic circuit provided with coils (Bilgin, Jiang, and Emadi 2019; Soares and Costa Branco 2001) as shown in Figure 3. In such a machine, a torque is produced due to the variable reluctance in the air gap between the rotor and the stator. The SRM motion is thus produced by the tendency to its rotor to move to a position where the reluctance of the excited winding is minimised (Bilgin, Jiang, and Emadi 2019; Soares and Costa Branco 2001). Then, the machine rotates the magnetic field by synchronising continuously and consecutively the supply of different phases with the rotor position by means of the firing angles (turn-on and turn-off). The SRM can be used as a motor or generator. The motoring mode is obtained by exciting the stator phase when the rotor is moving from the position where the air-gap is very big (shown in Figure 4a) towards the position where the air-gap is very small (given in Figure 4b), whereas the generating mode will be attained by exciting a stator phase when the rotor is moving from aligned position (given in Figure 4b) towards unaligned position (shown in Figure 4a) (Kadi and Brouri 2017; Emadi 2005; Laila and Adil 2019). To supply the stator phases of the SRM, an asymmetric half-bridge converter is the most adopted as can be seen in Figure 5. In the present paper, the used machine is characterised by a structure of 4 phases 8/6 (N_s = 8 is stator poles number and N_r = 6 is rotor poles number). Both stator and rotor magnetic circuits are made of M19, 29 gage silicon steel whose B-H characteristic is shown in Figure 6. The geometrical shapes of the studied SRM are given in Figure 7.

3. Equations describing the SRM

It is interesting to note that, mutual fluxes have a very weak value (Laila and Adil 2019), the mutual coupling between the adjacent phases can thus be neglected. Let i, R, L, ω, θ, and v denote the phase current, phase resistance, inductance, rotor speed, rotor position angle and phase voltage respectively.

Then, the voltage equation of a phase winding can be given as:

\[ v(t) = Ri + \frac{d\lambda(\theta, i)}{dt} \]  
(1)

where \( \lambda \) is the flux linkage per phase that can be expressed under the hypothesis of nonmagnetic coupling between phases, by:

\[ \lambda(\theta, i) = L(\theta, i) i \]  
(2)

Due to the nonlinear behaviour of the ferromagnetic material, the flux in the stator phases varies nonlinearly according to the rotor position \( \theta \) and the current \( i \) of each phase. Accordingly, through a derivation of (2) in (1), one immediately gets the following expression of the phase voltage:

\[ v(t) = Ri + L(\theta, i) \frac{di}{dt} + i \left( \frac{\partial L(\theta, i)}{\partial i} \frac{di}{dt} + \omega \frac{\partial L(\theta, i)}{\partial \theta} \right) \]  
(3)

The last term in (3) is the back electromotive force (EMF) voltage that is expressed by:
\[ e = \omega i \frac{\partial L(\theta, i)}{\partial \theta} \]  

\[ (4) \]

4. Inductance parameters determination of SRM

The aim presently is to develop an identification scheme allowing the inductance parameters determination of the SRM. Knowing that the nonlinear inductance \( L(\theta, i) \) depends on the rotor position \( \theta \) and the phase current \( i \). Then, for any fixed rotor position \( \theta \) (standstill test), the phase inductance \( L(i)|_\theta \) varies according to the current \( i \).

4.1. Nonlinear system describing the machine at standstill

Knowing that the back electromotive force (EMF) voltage is directly proportional to shaft speed (see (4)), this term is nil at standstill test. It follows that the phase voltage in (3) can be simplified as:

\[ v(t) = Ri + L(\theta, i) \frac{di}{dt} + i \frac{\partial L(\theta, i)}{\partial i} \frac{di}{dt} \]  

\[ (5) \]

Presently, the SRM rotor is blocked at a specific position with respect to phase to be tested, e.g. at aligned, unaligned and midway positions between the aligned and unaligned positions (see Figure 4). It is worth emphasising that the self-inductance of the stator phase at specific positions (in blocked rotor test) \( L(i)|_\theta \) varies with respect to the phase current as a smooth and continuous function (Figure 8). Then, the nonlinearity \( L(i)|_\theta \) can be described using a finite orthogonal function, e.g. polynomial decomposition (Sofiane, Tounzi, and Piriou 2002; Zhao et al. 2011):

\[ L(i)|_\theta = \sum_{k=0}^{n} a_k i^k \]  

\[ (6) \]
Where \( n \) is the polynomial function degree and \( a_k \) its coefficients. In this respect, note that most of previous works use polynomial approximation with degrees higher than \( n = 4 \) (Sofiane, Tounzi, and Piriou 2002). In the present study, we look for an optimal value for the polynomial degree \( n \) comparing the obtained results for different values of \( n \) (\( n = 3, 4, 5, 6, \) and 7). It is shown that \( n = 4 \) is a suitable choice. On the other hand, it readily follows from (5) and (6) that, the phase voltage can be expressed as:

\[
v(t) = R i(t) + \frac{d}{dt} \left( a_0 + \sum_{k=1}^{n} t^k (a_k + k a_k) \right)
\]

(7)

Accordingly, one immediately gets:

\[
v(t) = \frac{d}{dt} \left( \sum_{k=0}^{n} b_k t^{k+1} \right)
\]

(8a)

where:

Figure 7. The geometrical shapes of the SRM.
increases. In this respect, it is appropriate to develop an algorithm that converges substantially faster and possesses high identification accuracy. Within this context, an identification solution is suggested. Then, the phase winding is excited by a signal (current phase $i$) having a spectrum rich in frequency (see Figure 9).

Note that, the induced phase voltage is accessible to measurement. Presently, the SRM modelling and parameters identification is based upon sampling the input and output system (Figure 10). The sampling interval $T_s$ must satisfy the following hypothesis (Bai 2003):

$$T_s < \frac{2\pi}{M\omega_m} \quad \text{where} \quad M > 2$$

where $\omega_m$ is the maximal frequency of the input signal. For convenience, the discretisation form of (8a) is given as:

$$v(t) = \frac{\sum_{k=0}^{n} b_k (i(t)^{k+1} - i(t-1)^{k+1})}{T_s}$$

It is shown that the SRM (in standstill test) can be described by a nonlinear system of Hammerstein model. The parameters of the nonlinear model can be identified using data acquisition and the RLS algorithm. Let $\theta = [b_0 \ldots b_n]^T$ be the parameters vector, where the polynomial degree is chosen equal to 4. Then, the number of unknown parameters at any given position is 5 (see (8a)). Accordingly, the phase voltage given in (11) can be rewriting in the following regression form:
Let \( \phi \) be obtained.

Figure 4.3

\( J(\theta) = \sum_{k=1}^{N} (v(t) - \hat{v}(t))^2 = \sum_{k=1}^{N} (v(t) - \theta^T(t) \varphi(t))^2 \)  

where \( N \) is the data-length. Using the regression form (12a-b) and minimising the quadratic criterion \( J(\theta) \), an accurate estimate of the SRM parameters can be obtained. This can be achieved using the following steps:

Step 1: Initialisation
To initialise the algorithm, we set:
\[ \theta(0) = 1_{n \times 1}/\alpha \]  

\( P_n(0) = \alpha I_n \)  

for some large enough positive number \( \alpha \) and \( I_n \) is the identity matrix of size \( n \). Let take: \( \alpha = 10^6 \) and \( t = 1 \).

Step 2: Data-acquisition
Collect the input and output data (i.e. the current \( i(t) \) and voltage phase \( v(t) \)).

Step 3:
The information vector \( \varphi(t) \) can be established using (12b). Then, we compute the gain vector using the following expression:
\[ K_n(t) = \frac{P_n(t-1)\varphi(t)}{1 + \varphi^T(t)P_n(t-1)\varphi(t)} \]  

Update the covariance matrix:
\[ P_n(t) = P_n(t-1)(1 - K_n(t)\varphi^T(t)) \]  

Step 4:
Compute the following intermediate variable (estimation error):
\[ \epsilon(t) = v(t) - \varphi^T(t)\theta(t-1) \]  

Step 5: Parameters estimation
It follows (16)-(18), the parameter vector estimate of the SRM can be updated as:
\[ \theta(t) = \theta(t-1) + K_n(t)\epsilon(t) \]  

Step 6: if \( t = N \) go to end step, else increase the time by \( 1 \) \( (t = t + 1) \) and go to step 2.
End step.

5. Simulation results and discussion
To ensure the effectiveness of the proposed model, a 8/6 SRM is studied (Figures 7, Figures 11). Then, \( \theta = 30^\circ \) corresponds to the unaligned position of phase A (Figure 4a) at this position the inductance can be
handled as a constant (Fahimi et al. 1998, May), θ = 0° is chosen as the aligned position of phase A (Figure 4b), and θ = 15° corresponds to the midway position of phase A (Figure 4c). However, the data acquisition can be reached from experiments or FEM (Finite Element Method) technique, the FEM is used for this study. Accordingly, in the proposed method, the SRM is excited by a signal (current source) having rich frequency spectrum (e.g. multisine signal) at several specific positions (at aligned, midway, and unaligned position). Knowing the nonlinear model describing the SRM (Hammerstein structure), its parameters can then be estimated using data acquisition (i.e. current and voltage winding values) and recursive least square (RLS) (see step1 to 6 of the algorithm). Furthermore, the validation of the identification was performed using magnetostatic method (Kadi and Broui 2017) and experimental method (Figure 11).

Let us recall that, the magnetostatic method is a pure numerical ones which requires a full set of data obtained from FEM, its differs substantially of our method which based only upon the input-output measurement data (current and voltage).

From the achieved results given by Figures 12–16 we can draw the following remarks:

- When the air-gap is very small (at aligned position), the self-inductance is maximum. Similarly, when the stator and rotor poles are unaligned (the air-gap is very big) the self-inductance is minimal.

- It was also shown that these results of identification method allow to a reasonably good agreement between the self-inductance profiles using the proposed analytical approach and those obtained using magnetostatic and experimental methods.

- From the achieved curves, it is seen that the inductance at unaligned position, where the air-
gap is significant (Figure 4a), can be handled as constant due to less effect of saturation (Figures 14 and Figures 15). On the contrary, in the aligned position, where the air-gap is smaller (Figure 4b), the saturation effect is considerable, as can be seen (Figures 12 and Figures 15) the inductance decreases when the current increases. At the midway position the air gap is a little small (Figure 4b) and the saturation is also important (Figures 13 and Figures 15).

- The degree of magnetic saturation is high in the aligned position, where the air-gap is very small (Figure 4b), which results in a strong nonlinearity in the inductance profile.

As mentioned, the validation of the proposed approach is made by the experimental method as well as the magnetostatic ones. Then, the inductance as a function of the rotor position for a fixed current of 4A was determined and recorded by the analytical and experimental method. The estimated curve of the inductance is nearly close to the real one as shown in Figure 16. Furthermore, the values of the inductance for the proposed model and the experimental one in

![Figure 13. Self-inductance vs. current at midway position.](image1)

![Figure 14. Self-inductance vs. current at unaligned position.](image2)
the aligned position and unaligned position are close to the maximum and minimum values, respectively.

Figures 17–19 show the estimated parameters according to time. As can be seen, the used RLS algorithm in this study provides quicker convergence of parameters.

Then, note that the present analytical solution based on block-oriented identification of SRM (Hammerstein model) and using recursive least square gives very close results with those obtained using magnetostatic and experimental methods. Furthermore, it is seen that the proposed method requires a less computational load and minimal memory usage. Finally, the obtained results show that, it is not feasible to represent the switched reluctance machine by a linear model.

6. Conclusion

Switched reluctance machine is considered as highly nonlinear behaviour, which makes the SRM modelling difficult to achieve. One key step in the design of such scheme is the construction of the nonlinear system structured in blocks. Specifically,
Figure 17. Evolution of inductance parameters at aligned position.

Figure 18. Evolution of inductance parameters at midway position.

Figure 19. Evolution of inductance parameters at unaligned position.
it is shown that the SRM can be described by a Hammerstein model. To our knowledge, no previous work have been dealt with this technique. Then, an analytical identification method is developed to obtain the electrical parameters of SRM. The parameters identification is dealt based on a Recursive Least Square (RLS) algorithm. The originality of the present study lies in the fact that the machine model is represented using block-oriented systems. The proposed Hammerstein model still be very simple in use and the parameter identification can be easily achieved using RLS algorithm. Another main contribution of this paper is the design of recursive least square algorithm turned out to be consistent. Another feature of this work is the fact that the modelling design and identification scheme require a less computational load and low memory usage unlike several previous studies. The accuracy of the model has been verified by comparing the predicted characteristics with magnetostatic and experimental results.

In spite of all efforts in modelling and identification of SRM, finding an analytical model represent the nonlinear behaviour of SRM, is wide open for scientific research.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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